

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

Use this space for computations.

1 A parallelogram must be a rectangle when its

- (1) diagonals are perpendicular
- (2) diagonals are congruent
- (3) opposite sides are parallel
- (4) opposite sides are congruent

2 If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles not be congruent?

- (1) reflection over the x -axis
- (2) translation to the left 5 and down 4
- (3) dilation centered at the origin with scale factor 2
- (4) rotation of 270° counterclockwise about the origin

3 If the rectangle below is continuously rotated about side w , which solid figure is formed?



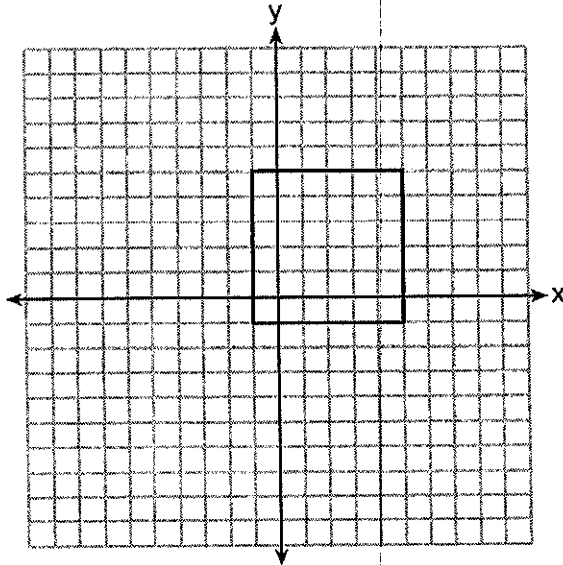
- (1) pyramid
- (2) rectangular prism
- (3) cone
- (4) cylinder

4 Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

- (1) $\cos(90^\circ - x)$
- (2) $\cos(45^\circ - x)$
- (3) $\cos(2x)$
- (4) $\cos x$

5 In the diagram below, a square is graphed in the coordinate plane.

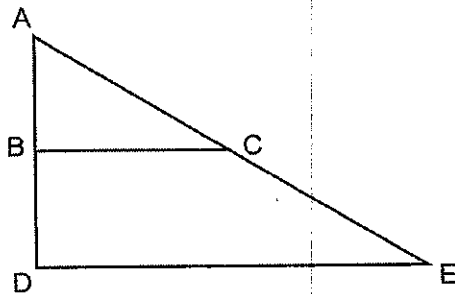
Use this space for computations.



A reflection over which line does not carry the square onto itself?

- (1) $x = 5$
- (2) $y = 2$
- (3) $y = x$
- (4) $x + y = 4$

6 The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below.



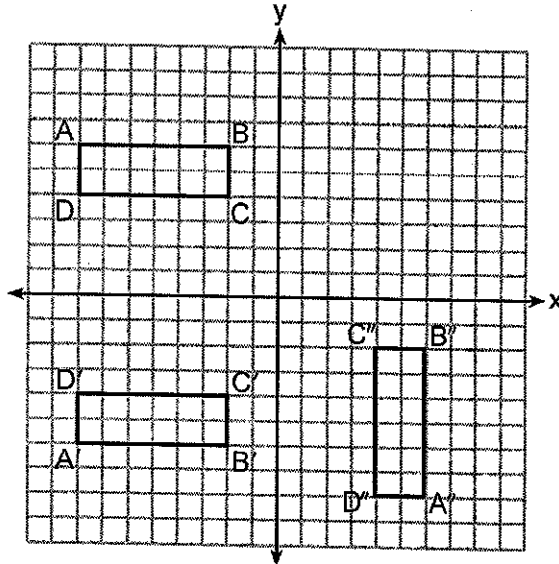
Which statement is always true?

- (1) $2AB = AD$
- (2) $\overline{AD} \perp \overline{DE}$
- (3) $AC = CE$
- (4) $\overline{BC} \parallel \overline{DE}$

dilation preserves parallelism

7 A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.

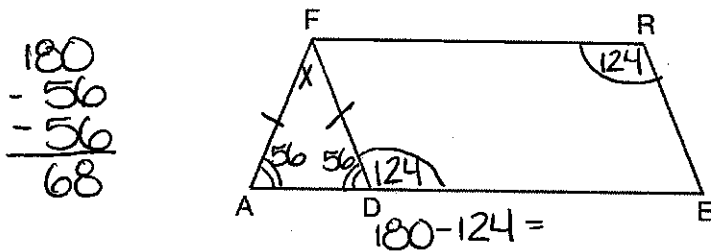
Use this space for computations.



Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

- a reflection followed by a rotation
- (2) a reflection followed by a translation
- (3) a translation followed by a rotation
- (4) a translation followed by a reflection

8 In the diagram of parallelogram $FRED$ shown below, \overline{ED} is extended to A , and \overline{AF} is drawn such that $\overline{AF} \cong \overline{DF}$.



If $m\angle R = 124^\circ$, what is $m\angle AFD$?

- (1) 124°
- (2) 112°
- 68°
- (4) 56°

9 If $x^2 + 4x + y^2 - 6y - 12 = 0$ is the equation of a circle, the length of the radius is

- (1) 25
 (2) 16
 5
 (4) 4

$$\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$$

$$\left(\frac{4}{2}\right)^2 = 2^2 = 4$$

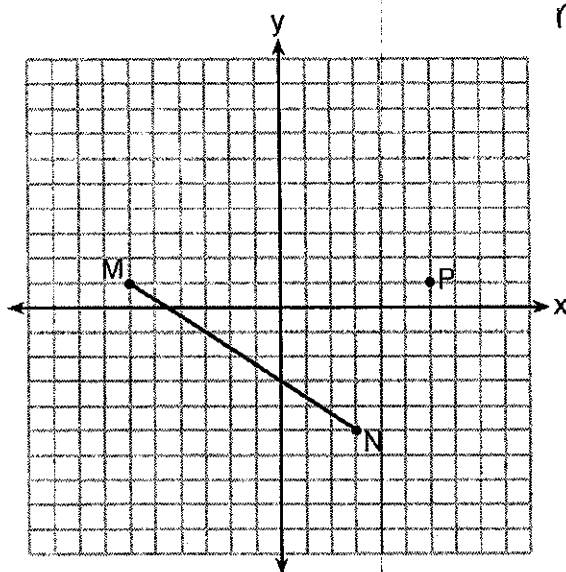
Use this space for computations.

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$$

$$(x+2)^2 + (y-3)^2 = 25$$

$$r = 5$$

10 Given \overline{MN} shown below, with $M(-6,1)$ and $N(3,-5)$, what is an equation of the line that passes through point $P(6,1)$ and is parallel to \overline{MN} ?



$$m = \frac{1 - (-5)}{-6 - 3} = \frac{6}{-9}$$

$$y - 1 = \frac{6}{-9}(x - 6)$$

$$y - 1 = -\frac{2}{3}x + 4$$

$$y = -\frac{2}{3}x + 5$$

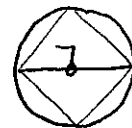
$y = -\frac{2}{3}x + 5$ (3) $y = \frac{3}{2}x + 7$

(2) $y = -\frac{2}{3}x - 3$ (4) $y = \frac{3}{2}x - 8$

11 Linda is designing a circular piece of stained glass with a diameter of 7 inches. She is going to sketch a square inside the circular region.

To the nearest tenth of an inch, the largest possible length of a side of the square is

- (1) 3.5 (3) 5.0
 4.9 (4) 6.9



$$x^2 + x^2 = 7^2$$

$$2x^2 = 49$$

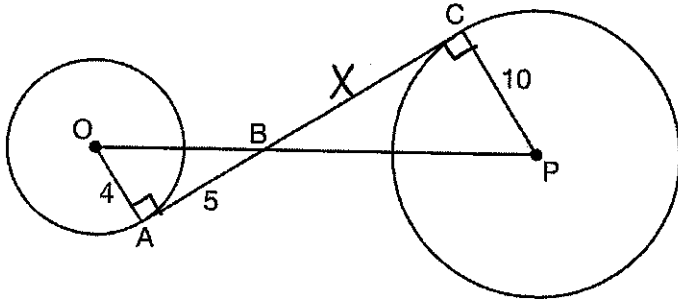
$$x^2 = 24.5$$

$$x = 4.9$$

[OVER]

- 12 In the diagram shown below, \overline{AC} is tangent to circle O at A and to circle P at C , \overline{OP} intersects \overline{AC} at B , $OA = 4$, $AB = 5$, and $PC = 10$.

Use this space for computations.



$$\frac{4}{5} = \frac{10}{x}$$

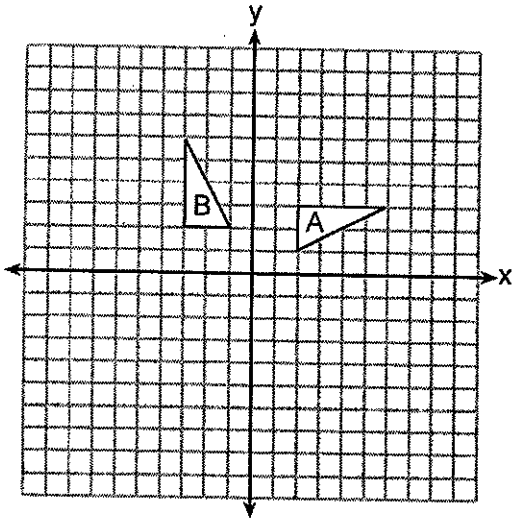
$$4x = 50$$

$$x = 12.5$$

What is the length of \overline{BC} ?

- (1) 6.4
- (2) 8
- (3) 12.5
- (4) 16

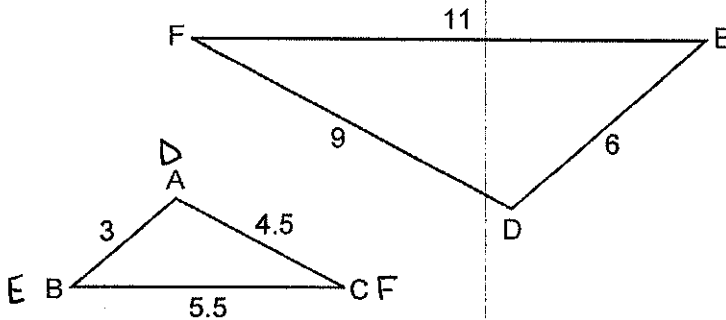
- 13 In the diagram below, which single transformation was used to map triangle A onto triangle B ?



- (1) line reflection
- (2) rotation
- (3) dilation
- (4) translation

Use this space for computations.

14 In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.



Which relationship must always be true?

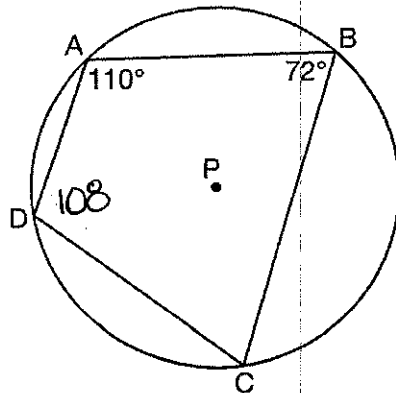
(1) $\frac{m\angle A}{m\angle D} = \frac{1}{2}$

(3) $\frac{m\angle A}{m\angle C} = \frac{m\angle F}{m\angle D}$

(2) $\frac{m\angle C}{m\angle F} = \frac{2}{1}$

$\frac{m\angle B}{m\angle E} = \frac{m\angle C}{m\angle F}$

15 In the diagram below, quadrilateral $ABCD$ is inscribed in circle P .



What is $m\angle ADC$?

(1) 70°

108°

(2) 72°

(4) 110°

$\angle B + \angle D = 180$
 $72 + x = 180$

16 A hemispherical tank is filled with water and has a diameter of 10 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank, to the nearest pound?

- (1) 16,336
- (2) 32,673
- (3) 130,690
- (4) 261,381

Use this space for computations.

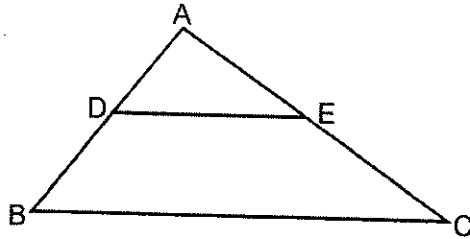
$$D = 10$$

$$r = 5$$

$$V = \frac{1}{2} \cdot \frac{4}{3} \pi (5)^3$$

$$V = 261.7994 (62.4) = 16336$$

17 In the diagram below, $\triangle ABC \sim \triangle ADE$.



$$\frac{AD}{AB} = \frac{AE}{AC}$$

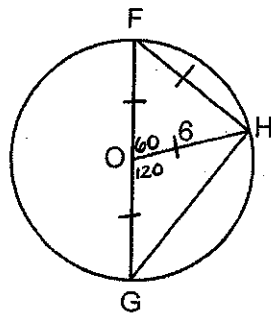
Which measurements are justified by this similarity?

- (1) $AD = 3, AB = 6, AE = 4,$ and $AC = 12$
- (2) $AD = 5, AB = 8, AE = 7,$ and $AC = 10$
- (3) $AD = 3, AB = 9, AE = 5,$ and $AC = 10$
- (4) $AD = 2, AB = 6, AE = 5,$ and $AC = 15$

$$\frac{2}{6} = \frac{5}{15}$$

$$30 = 30$$

18 Triangle FGH is inscribed in circle O , the length of radius \overline{OH} is 6, and $\overline{FH} \cong \overline{OG}$.



$$\frac{60}{360} \cdot \pi r^2$$

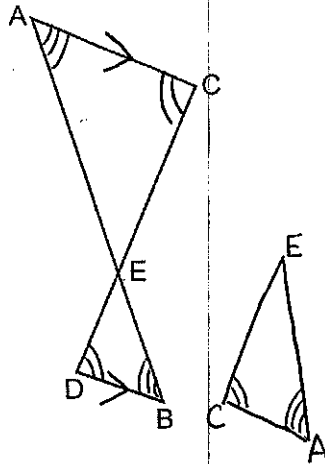
$$\frac{1}{6} \cdot 6^2 \pi$$

$$\frac{1}{6} \cdot 36 \pi$$

What is the area of the sector formed by angle FOH ?

- (1) 2π
- (2) 3π
- (3) $\frac{3}{2}\pi$
- (4) 24π

19 As shown in the diagram below, \overline{AB} and \overline{CD} intersect at E , and $\overline{AC} \parallel \overline{BD}$.



Given $\triangle AEC \sim \triangle BED$, which equation is true?

(1) $\frac{CE}{DE} = \frac{EB}{EA}$

(3) $\frac{EC}{AE} = \frac{BE}{ED}$

(2) $\frac{AE}{BE} = \frac{AC}{BD}$

(4) $\frac{ED}{EC} = \frac{AC}{BD}$

20 A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

- (1) The area of the image is nine times the area of the original triangle.
- (2) The perimeter of the image is nine times the perimeter of the original triangle.
- (3) The slope of any side of the image is three times the slope of the corresponding side of the original triangle.
- (4) The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

21 The Great Pyramid of Giza was constructed as a regular pyramid with a square base. It was built with an approximate volume of 2,592,276 cubic meters and a height of 146.5 meters. What was the length of one side of its base, to the nearest meter?

(1) 73

(3) 133

(2) 77

(4) 230

Use this space for computations.

side = perimeter
 $(\text{side})^2 = \text{area}$

$$V = \frac{1}{3} Bh$$

$$2592276 = \frac{1}{3} S^2 (146.5)$$

$$\frac{2592276}{1} = \frac{146.5 S^2}{3}$$

[OVER]

$$7776828 = 146.5 S^2$$

$$\sqrt{53084.15017} = \sqrt{S^2}$$

$$230 = S$$

22 A quadrilateral has vertices with coordinates $(-3,1)$, $(0,3)$, $(5,2)$, and $(-1,-2)$. Which type of quadrilateral is this?

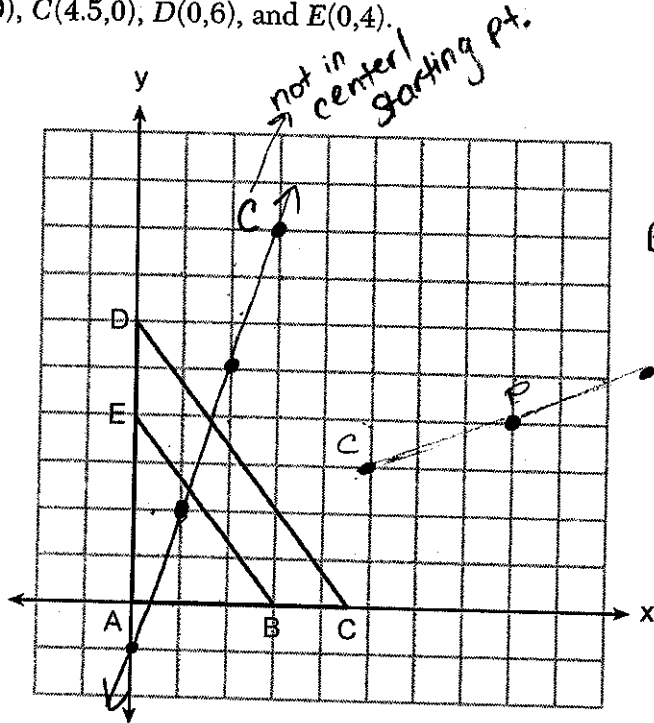
- (1) rhombus
 (2) rectangle
 (3) square
 (4) trapezoid

A B C
 $AB = \frac{3-1}{0-3} = \frac{2}{3}$
 $BC = \frac{3-2}{0-5} = \frac{1}{-5}$

Use this space for computations.

$CD = \frac{2-2}{5-1} = \frac{0}{4} = 0$
 $DA = \frac{-2-1}{-1-3} = \frac{-3}{-4} = \frac{3}{4}$

23 In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.



$BE = \sqrt{(3-0)^2 + (0-4)^2}$
 $= \sqrt{3^2 + 4^2}$
 $= \sqrt{25}$
 $= 5$

$CD = \sqrt{(4.5-0)^2 + (0-6)^2}$
 $= \sqrt{4.5^2 + 6^2}$
 $= \sqrt{56.25}$
 $= 7.5$

The ratio of the lengths of \overline{BE} to \overline{CD} is

- (1) $\frac{2}{3}$
 (2) $\frac{3}{2}$
 (3) $\frac{3}{4}$
 (4) $\frac{4}{3}$

$\frac{5}{7.5} = \frac{2}{3}$

24 Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3,8)$. The line's image is

- (1) $y = 3x - 8$
 (2) $y = 3x - 4$
 (3) $y = 3x - 2$
 (4) $y = 3x - 1$

*see graph above

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the *nearest thousandth*.

State which type of wood the cube is made of, using the density table below.

Ash

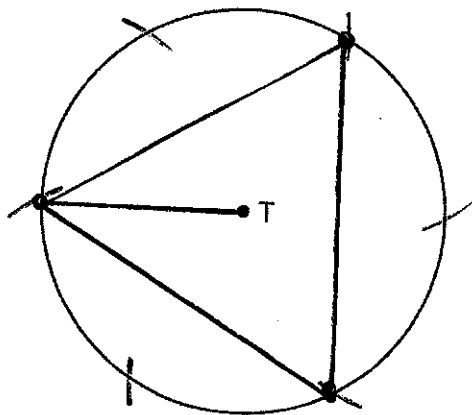
$$V = 6(6)(6) = 216$$

$$D = \frac{M}{V} = \frac{137.8}{216}$$

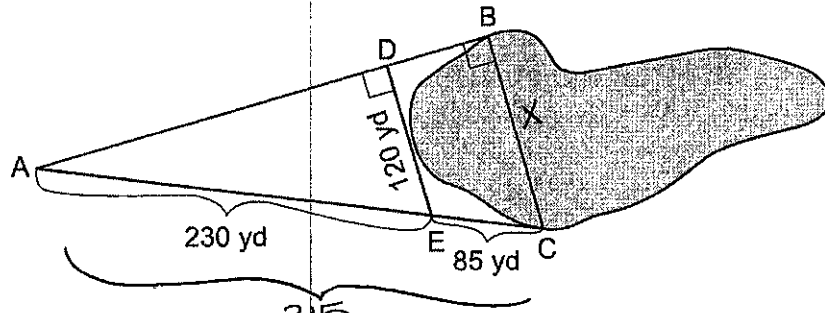
$$D = .638$$

Type of Wood	Density (g/cm ³)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
* Ash	0.638
Maple	0.676
Oak	0.711

- 26 Construct an equilateral triangle inscribed in circle T shown below.
[Leave all construction marks.]



27 To find the distance across a pond from point B to point C , a surveyor drew the diagram below. The measurements he made are indicated on his diagram.



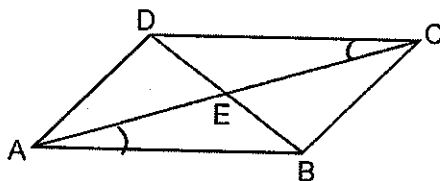
Use the surveyor's information to determine and state the distance from point B to point C , to the nearest yard.

$$\frac{\text{hyp}}{\text{leg}} \quad \frac{230}{120} = \frac{315}{X}$$

$$230X = 37800$$

$$\boxed{X = 164}$$

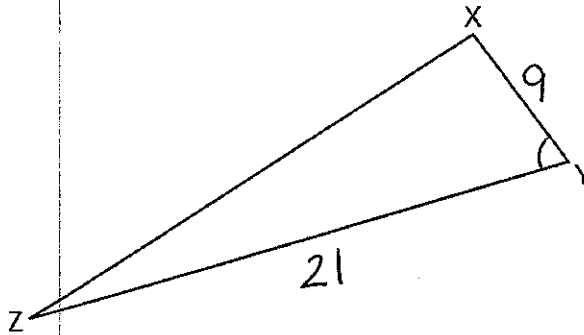
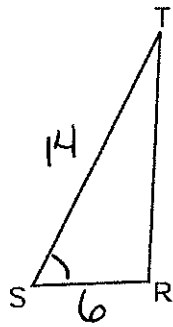
28 In parallelogram $ABCD$ shown below, diagonals \overline{AC} and \overline{BD} intersect at E .



Prove: $\angle ACD \cong \angle CAB$

S	R
① Parallelogram $ABCD$ ② $\overline{AB} \parallel \overline{DC}$ ③ $\angle ACD \cong \angle CAB$	① Given ② In a parallelogram opposite sides are \parallel ③ \parallel lines cut by a transversal form \cong alternate interior \angle s

29 Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.



$$\frac{14}{6} = \frac{21}{9}$$

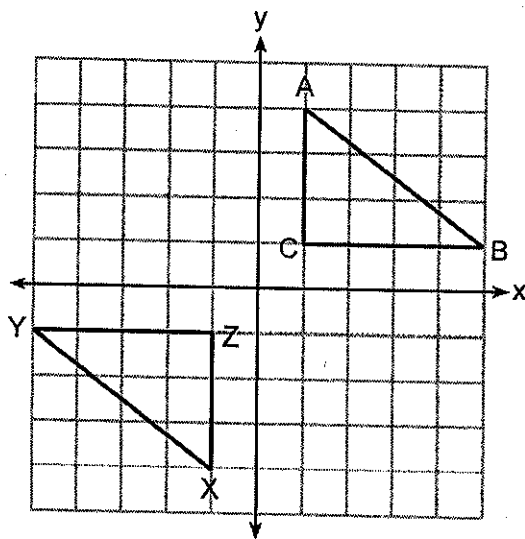
$$126 = 126$$

✓

corresponding sides are in proportion

SAS similarity

30 In the diagram below, $\triangle ABC$ and $\triangle XYZ$ are graphed.



Use the properties of rigid motions to explain why $\triangle ABC \cong \triangle XYZ$.

A rotation of 180° around the origin maps
point A onto X
B onto Y
C onto Z

31 The endpoints of \overline{DEF} are $D(1,4)$ and $F(16,14)$. Determine and state the coordinates of point E , if $DE:EF = 2:3$.

$$\frac{2}{5}$$

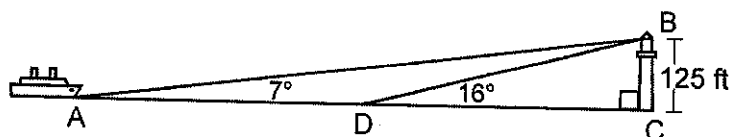
$$D(1,4) \xrightarrow{T_{15,10}} F(16,14)$$

$$D(1,4) \xrightarrow[\begin{matrix} T_{6,4} \\ T_{\frac{2}{5}(15), \frac{2}{5}(10)} \end{matrix}]{\quad} \boxed{E(7,8)}$$

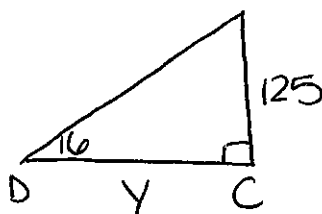
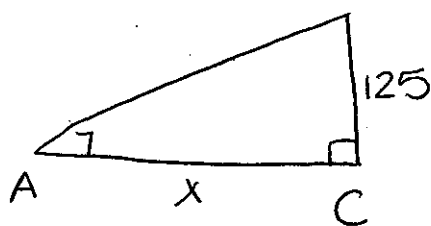
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 32 As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7° . A short time later, at point D, the angle of elevation was 16° .



To the nearest foot, determine and state how far the ship traveled from point A to point D.



$$\frac{\tan 7}{1} = \frac{125}{x}$$

$$.1228x = 125$$

$$x = 1018.0433$$

$$\frac{\tan 16}{1} = \frac{125}{y}$$

$$.2867y = 125$$

$$y = 435.9268$$

$$AC - DC = AD$$

$$1018.0433 - 435.9268$$

$$\boxed{582}$$

33 Triangle ABC has vertices with $A(x,3)$, $B(-3,-1)$, and $C(-1,-4)$.

Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle.

[The use of the set of axes below is optional.]

$$AB = \frac{3 - (-1)}{x - (-3)} = \frac{4}{x+3}$$

OR $BC = \frac{3}{-2}$

$$BC = \frac{-1 - (-4)}{-3 - (-1)} = \frac{3}{-2}$$

$$CA = \frac{3 - (-4)}{x - (-1)} = \frac{7}{x+1}$$

$$\boxed{AB \perp BC}$$

\perp lines form right \angle s and a right \triangle has 1 right \angle

$$\frac{4}{x+3} = \frac{2}{3}$$

$$2x + 6 = 12$$

$$2x = 6$$

$$\boxed{x = 3}$$

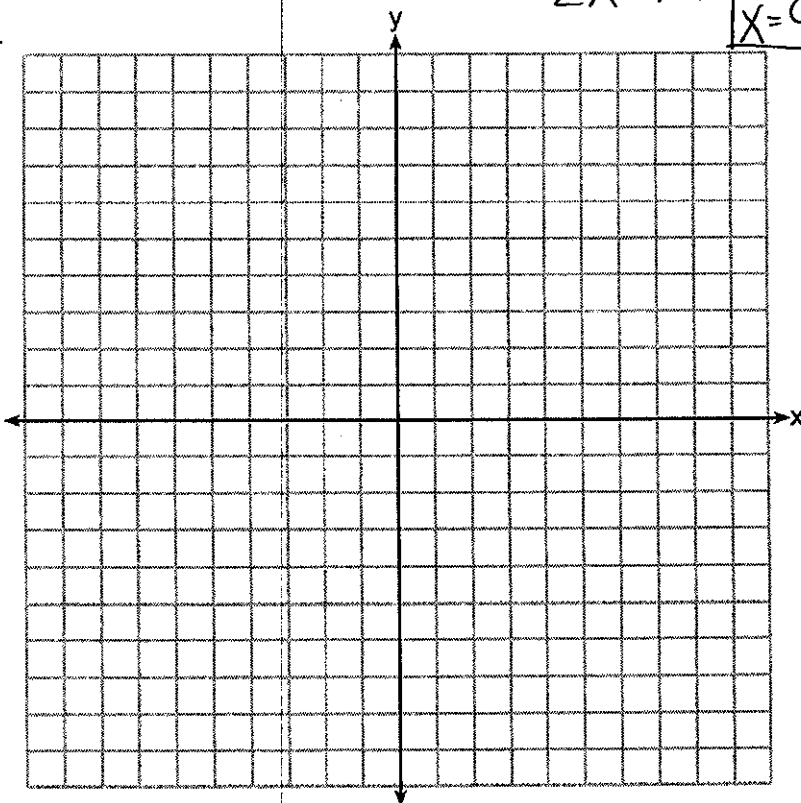
$$\boxed{BC \perp CA}$$

$$\frac{3}{-2} = \frac{7}{x+1}$$

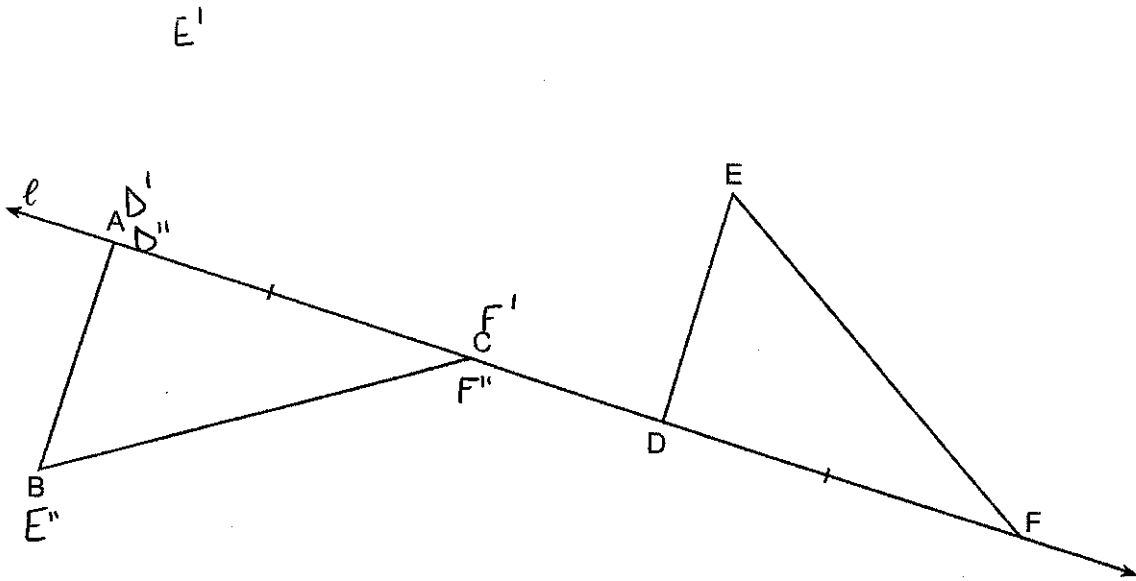
$$2x + 2 = 21$$

$$2x = 19$$

$$\boxed{x = 9.5}$$



34 In the diagram below, $\overline{AC} \cong \overline{DF}$ and points A , C , D , and F are collinear on line ℓ .



Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along ℓ , such that point D is mapped onto point A . Determine and state the location of F' . Explain your answer.

F maps onto C (which is F') under a translation

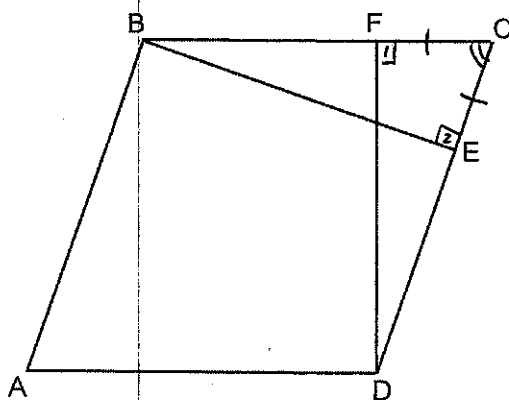
Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line ℓ . Suppose that E'' is located at B . Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

Yes. A reflection is a rigid motion that preserves length/distance and angle measure

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 In the diagram of parallelogram $ABCD$ below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, and $\overline{CE} \cong \overline{CF}$.



Prove $ABCD$ is a rhombus.

S	R
① Parallelogram $ABCD$ $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$	① Given
② $\angle 1 \cong \angle 2$	② \perp lines form \cong right \angle s
③ $\angle C \cong \angle C$	③ Reflexive
④ $\triangle CEB \cong \triangle CFD$	④ ASA \cong ASA
⑤ $\overline{CB} \cong \overline{CD}$	⑤ CPCTC
⑥ $ABCD$ is a rhombus	⑥ A rhombus is a parallelogram with consecutive \cong sides

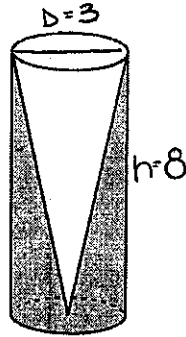
36 Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the nearest cubic inch, what will be the total volume of 100 candles?

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{1}{3}\pi (1.5)^2 (8)$$

$$18.8496 (100)$$

$$\boxed{1885 \text{ in}^3}$$



Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?

$$1885(.52) = 980.2 (\$.10) = \boxed{\$98.02}$$

If Walter spent a total of \$37.83 for the molds and charges \$1.95 for each candle, what is Walter's profit after selling 100 candles?

$$1.95(100) = 195$$

$$37.83 + 98.02 = 135.85$$

mold wax

$$195 - 135.85 = \boxed{\$59.15}$$

